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*Philosophy of the Social Sciences* 2009; 39; 507 originally published online Apr 16, 2009;
DOI: 10.1177/0048393109334584

**Philosophy of the Social Sciences**
Volume 39 Number 3
September 2009 507-526

10.1177/0048393109334584
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# The Advantage of Theft over Honest Toil

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Gregory Landini offers a new and an illuminating reading of Ludwig Wittgenstein's idea about his own innovation: it is the invention of a notation that removes the mystery from all theorems of logic and of mathematics as it renders their proofs part of their wordings. This makes all theorems in principle as boring as "all four-legged animals are animals." This idea is Wittgenstein's doctrine of showing. It is worthless; yet, as Landini shows, every time Wittgenstein offered an elaboration on it, Russell checked it carefully and found it of no value. This, let me add, shows that Russell was in error in suggesting that intellectually there is no "advantage of theft over honest toil": at times one may pay back and with high interest. Other cases may be due to misjudgment rather than to sloth.

***Keywords:*** *history of logic; logical positivism; Russell; Wittgenstein*

Landini, G. (2007). *Wittgenstein's Apprenticeship with Russell*. Cambridge: Cambridge University Press.

This remarkable book will fascinate those interested in the history of subtle problems of logic or in the forgotten details of the history of early twentieth-century philosophy, not to mention the rise of powerful logical analysis that is hopefully here to stay and the associated rise and fall of analytic philosophy. Yet it addresses specialized experts in the logical literature of the period. A proper reading of it requires much effort and more checking. This is not necessary for readers of this journal; my aim here shall be to report what might interest a broader audience, and to offer some background information and some minor critical comments.

Landini takes seriously the assertion of Ludwig Wittgenstein: "what a philosopher wants is . . . a new notation." His thesis is straightforward: there was never any ground for Wittgenstein's hope to find a notation that will solve all problems of philosophy and of logic by rendering their truths obvious. Bertrand

*Received* 27 August 2008

Russell's logical system seemed like a step in this direction, at least as Wittgenstein understood it. Wittgenstein wanted to go Russell's way all the way. Some esteemed commentators ignore this. They partake in a process of "marginalization of Russell" (p. 2) that marginalizes Wittgenstein too, although their intention was to inflate the value of the early work of Wittgenstein by playing down the vastness of Russell's influence on him. Landini presents Wittgenstein's system as interesting, as a tight system that follows his central philosophical idea, but as a nonstarter. Wittgenstein's aficionados now have to decide: do they prefer to view his message as uninteresting or as an interesting but failed project?

Wittgenstein's central idea[1] is his doctrine of showing: the truths of logic and of metaphysics are beyond the reach of human language. Many of his fans admired him for his alleged contempt for metaphysics. Landini sidesteps this folly. Rather, he centers on logic. Wittgenstein said, "Laws of inference . . . are senseless and superfluous" (*Tractatus*, 5.132). Many commentators join Russell's dismissal of this ban on the use of the meta-language. Yet Russell spent much effort examining carefully the logic in which Wittgenstein spells out this untenable doctrine. This fact engages Landini here.

How much does a study of historical details alter our view of the broad outline of history? We do not know.[2] Their corrections, we hope, reflect more inner rationality than the mere waywardness of fashion; the merely fashionable, we hope, drops out of the broad outline faster than the reasonable. How do we judge these? Views of rationality also change. Realizing this we may feel at a loss—unless we suggest that intuitions of reasonableness and criteria for them do not easily sway in the wind of wayward fashion.

Some commonsense rules of exegesis should suffice. The popular understanding of a text soon after its appearance takes precedence over the better understanding of it, but only as the default option; a better understanding may prevail later. We take for granted, *pace* Jacques Derrida

1. Wittgenstein's doctrine of showing plays a big role in Max Black's *A Companion to Wittgenstein's Tractatus* (1964). That book is defensive and not as extensive and systematic as the present work. Russell rejected the doctrine (p. 216). He once spoke of Wittgenstein in allusion to the doctrine of showing in a very obscure pun that must be a private joke: "He, himself, as usual, is oracular and emits his opinion as if it were a Czar's ukase, but humbler folk can hardly content themselves with this procedure" (Russell 1959, 88). "Ukase" denotes any edict that Czars utter, etymologically it means "show" or "point at."

2. Thomas S. Kuhn declared that research leaders decide when the accumulation of incongruities of the unique central idea of physical research of the age is sufficiently large to warrant a switch to a new central idea and a new age. Too many historians in Kuhn's vicinity were Hegelians, many of them unawares, and unawares he joined them.

(perhaps in a vulgar reading), that not all readings are equal. We may judge not too reasonable much traditional orthodox exegesis (say, of biblical texts according to an orthodoxy that we do not share); this separates the authoritarian reading from critical efforts to read texts historically; *pace* Derrida (perhaps in a vulgar reading), we prefer critical–historical readings as the more rational, at least at first blush: we often consider rational our endorsement of new, corrected readings of texts. Nevertheless, we do not forget all erroneous readings:[3] some errors are parts of our heritage. Some of these enter the broad outline of our history; others drop out. Having no criterion, we usually follow the idea that a story wants some inner logic. The inner logic too may alter. And, again, we hope that such changes exhibit some inner reasonableness. We usually try to find the advantages of a more accurate reading of a text over a superficial one, and we hope that the reading that is more accurate offers a deeper understanding. This has a natural limit. We judge most texts not worthy of efforts to interpret them.[4] And in whatever reading we offer of even the best texts extant, it is still wanting: no text is fully satisfactory in any reading. For a reading to be historical (rather than pious), it should not plant into a text ideas that became known only later.

Exceptionally, Landini adheres to these commonsense rules. He reports the traditional reading of the story[5] and corrects it.[6] I am afraid he sells his wares too cheaply. I find his presentation of logicism (p. 147ff.), for example, more interesting than his discussion of the question, was Wittgenstein a

3. No matter how wide of the mark readings of the Vienna Circle and other twentieth-century commentators may have been, they are parts of philosophical tradition. Items may drop out of tradition, though: philosophers and whole schools of philosophy did.

4. This holds for serious texts; alas, we are all too quick to express disdain for a text by preferring a superficial reading of it to a serious study of its details. Yet neglect is unavoidable, and the neglect that most Wittgenstein fans show of the technical part of his first book (that is the heart of Landini's exegesis) is understandable. It is disdain for hard work that they share with Wittgenstein, said Russell (pp. 249–50), who found their antimetaphysics facile.

5. The received readings of Russell are unusually problematic. His output was enormous, few read him and fewer comprehended him (since his writings are deceptively easy and simple: they are more technically difficult and more subtle than they look). Also, many of his better commentators offered their own variants of his ideas—usually as elucidations of his texts—in humility and good faith or in understandable efforts to rely on a great authority.

6. I am not qualified to judge the correctness of Landini's picture of the received reading of Wittgenstein, since the opinions regarding Wittgenstein's apprenticeship with Russell that he declares popular (and his criticism of it) are new to me. I learned about Wittgenstein very late in my education. As an undergraduate I took no course in logic but read Russell's *Principles* of 1903 and my access to logic was through the eyes of my mathematics and philosophy teachers—Abraham Fraenkel and Karl Popper, both quite outside the mainstream of logic.

logicist. Yet logicism belongs to an earlier book of Landini's; this one concerns Wittgenstein as a disciple of Russell—mainly two items about him. First is the widespread myth that Ray Monk has crystallized (1990): at a certain stage, the roles of Russell and of Wittgenstein as master and apprentice underwent reversal. Second is the new light that Russell's manuscript book (1912–1913) sheds on events. It is *Theory of Knowledge* that he wrote with high hopes and then suppressed under criticism of Wittgenstein. Both items involve intricate considerations and subtle details. These are fascinating; for the broad outline of the story, however, they barely signify, and their significance diminishes with two factors. First is the waning of Wittgenstein's influence. Second is the general retreat from Russell's search for a grand system that should incorporate the last word on logic, meta-mathematics, ontology, epistemology, and more. The book gives the sense that its author swims against the current. This seems to me a slight exaggeration, even though his contribution is new and instructive—at least in its detail and precision. It is this: Wittgenstein's first book is cryptic at least in part because he wrote it as an (alleged) improvement on *Principia Mathematica* of Whitehead and Russell and with antiphilosophy intended to appear as an imposing corollary. The importance of mathematical logic for the young Wittgenstein is obvious and yet obviously neglected.[7] Hence, the matter at hand concerns not pecking order but the right reading of Wittgenstein's notoriously cryptic text, the presentation of his logic as central to his discussion.

Landini's attention to detail is admirable. He studied carefully texts, from early Russell (1903) and Wittgenstein's first book (1922) to the latest exegesis on them. Yet he cannot possibly offer sufficiently precise versions of his arguments that often hinge on detailed and difficult logical studies. Hopefully, some of Wittgenstein's fans will test them as carefully and dispassionately as they require. Landini himself is obviously no Wittgenstein fan. He endorses Russell's view of his first book as "oracular" (pp. 192, 196, 200ff., 226, 231, 252): the book "promises solutions for the problems of philosophy. It is a disappointment that it offers only programmatic gestures toward solutions" (p. 189). He does so responsibly, after following in

7. Victor Rodych (2007) says of Wittgenstein's output on mathematics that it is

> undoubtedly the most unknown and underappreciated part of his philosophical opus. Indeed, more than half of Wittgenstein's writings from 1929 through 1944 are devoted to mathematics, a fact that Wittgenstein himself emphasized in 1944 by writing that his 'chief contribution has been in the philosophy of mathematics.'

In his conclusion Rodych expresses the hope that some future mathematicians will behave in accord with Wittgenstein's sentiment. He does not explain.

some detail the efforts to explicate these gestures that Russell (Introduction to the second edition of the *Principia*, 1925) and Ramsey (in his review of it) have made (chap. 6).

Consider first the assessment that Monk has crystallized. It has scarcely any text to lean on. The text Monk relies on is familiar,[8] since Russell generously acknowledged what he learned from Wittgenstein.[9] It was logical atomism. He refuted the version of it that Wittgenstein had advocated. (The information that a list of atomic sentences is complete or incomplete is obviously neither atomic nor compound and hence one that Wittgenstein forbade.) Wittgenstein never responded to his criticism: he never said whether he stuck to his logical atomism or let go of it.[10] Nor was it easy, as he offered it as a part of a tight-knit and stunningly simplified version of Russell's monumental system of logic. Russell dismissed Wittgenstein's system *holus bolus*. (It forbids most of mathematics; it manages to articulate much that it declared not given to articulation; and, above all, it is mystical.)[11] Wittgenstein then moved from logic to the philosophy of mathematics and to his resolute battle against the metaphysicians. His works on mathematics were never fashionable, but his antimetaphysics won him his great fame and an enormous following. Oddly, neither he nor his followers succeeded in any demolition job. Russell did, and his targets were specific metaphysical commitments that mathematics and logic managed to dispose of (especially the success of modern logic in breaking away from the metaphysical baggage attached to Aristotle's limitation of language to

8. Russell published Wittgenstein's logical atomism before Wittgenstein did, and with generous acknowledgement. He also reported disappointments about it. These Monk ignores. In Russell's 1940 *Inquiry* (p. 196) and in his 1959 *My Philosophical Development* (p. 249) we find reference to his failed efforts of two decades of work on Wittgenstein's ideas. Despite repeated disappointment, he never let go of the possibility that Wittgenstein was right on any item. He expressed his final disappointment at the very end of his obituary on Wittgenstein (1951). He later said that he dismissed the output of the later Wittgenstein (p. 249) as stemming from intellectual laziness.

9. It is advisable to disregard the unpleasant feel of Wittgenstein's references to Russell—his acknowledgement to "my friend Mr. Bertrand Russell" (*Tractatus*, Preface) and his letters to Russell with requests for small chores and those to others with reports on unwanted advice he had given Russell, not to mention his expressions of annoyance at Russell's refusal of the advice (p. 219). The same holds for his calling Russell's use of a procedure illicit (*Tractatus*, 5.452) and his calling by different names his own use of that same procedure.

10. The latest word on this is due to Jaakko Hintikka (1996): Wittgenstein clung to all details of his first book but one: the picture theory of language. See my review of it (Agassi 2000). Landini finds Wittgenstein "departing further and further" from claiming uniqueness for his technique (pp. 249–50).

11. As Wittgenstein writes down certain formulas and adds that they "cannot be written down" (*Tractatus,* 5.535), his meaning is definitely not literal. Yet he is the famous crusader for clarity.

categorical propositions). The only possible exception to this is his opposition to Gottlob Frege's Platonism, the view of concepts as existing outside space and time (on which he finally admitted defeat in his 1940 *Inquiry into Meaning and Truth*). He never endorsed Wittgenstein's idea that clarification destroys all (articulated) metaphysics. Even when he gave up certain metaphysical ideas, such as the theory of substance,[12] he did not see this as a clarification, even though evidently the growth of modern logic was a significant contributor to this change. This way Russell reduced the significance of logical atomism. This disposes of Monk's major evidence.

Monk finds much weightier evidence in Russell's Introduction to the second edition of the *Principia* (1925): capitulation to Wittgenstein, no less. And Landini disproves this by quoting a passage from that Introduction (p. 190).

Admittedly, some developments due to clarification Wittgenstein-style did occur. But they never were metaphysically significant except for Russell's dismissal of the concept of substance. The paradigm cases of clarification are of some expressions like Wittgenstein's example, "Socrates is identical" that are neither true nor false. Some philosophers sought rules to explain this. Frege declared that what is neither true nor false has no meaning (is a pseudo-sentence), and he argued that the sentence in question is such since "identical" is a relation, not a predicate. Russell went further and outlawed many expressions to outlaw his paradox.[13] He said, Plato's sentence "only beauty is beautiful" is a pseudo-sentence. The importance of this lay elsewhere: for the first time contradictions won full recognition as meaningful.[14] Russell went further and tried to eliminate in this way the Platonism of Frege's system. In this he had only partial success, but this sufficed to raise Wittgenstein's hope that logic allows for the elimination of all metaphysical assertions by clarifying them out of existence. He never proved this; careful people like Rudolf Carnap, the logician of the "Vienna Circle," vouched for him, saying that he did. (This is bewildering.)

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12. Russell's and Wittgenstein's discussions of the "unitary soul" (p. 195) is incomprehensible without reference to substance. Incidentally, the same holds already for Hume and for Mach (and any other advocate of neutral monism).

13. A paradox is a proven contradiction. Russell's paradox is this. Normally a class is not a member of itself. Consider then the class of normal classes and ask if it is normal. It turns out that this class is normal if it is not and vice versa.

14. Even such clear writers as Hume and Berkeley were unclear about contradictions, as they held the old doctrine of meaning as denotation and that inconsistent terms (like square-circular) and even empty ones (like "mermaid") denote nothing. Frege's refutation of the old doctrine of meaning and Russell's desire to get rid of his paradox enabled him to clarify the situation. Frege himself was still unclear about it.

The body of Wittgenstein's contribution to logic and to the philosophy of mathematics is marginal at best; this is the received opinion.[15] Nor is this opinion new: the "Vienna Circle" and other Viennese followers of Wittgenstein endorsed his idea that metaphysics is meaningless, not his rationale for it; they took for granted the variant of logic that Carnap had offered, and it opposed Wittgenstein explicitly.[16]

All this is received opinion in broad outline; it does not depend on Russell's manuscript book (*Theory of Knowledge*, 1912–1913) that Landini discusses, and it shows Monk's claim to be feeble hagiography (pp. 2–9): Landini's opponents are weak.

Landini shows that Russell's *Theory of Knowledge* is very interesting, at the very least as "revealing the unity of Russell's method" (p. 21), and he uses it to throw interesting new light on Russell as well as on Wittgenstein. In particular, he suggests that Wittgenstein's criticism of it was weak and that Russell fell back on its theory, perhaps in milder and more cautious versions,[17] and in full recognition that it was more of a program than of a finished product.[18]

Russell admitted clearly that his struggle in his effort to present a perfect system of logic was a failure.[19] As to Wittgenstein, he admitted that the

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15. See Rodych (2007).

16. Carnap advocated logic in a Russellian vein; his program to deny reality to sets is very similar to Russell's as Landini presents it (see next note). As Russell's effort was unsuccessful, Carnap attempted a compromise between Russell and Hilbert. This does not work. See Agassi 1988 and 2006.

17. Russell was famous for his ability to change his mind and although he never concealed his changes of opinion, his readers were not always able to keep up with him.

18. The manuscript was "largely buried in unpublished manuscripts." (Landini has devoted to it an earlier monograph, 1998.) It incorporates the refusal to ascribe reality to classes (p. 7); these comprise "logical fiction" (p. 19). Hence, Landini stresses, Russell meant his theory of types as applicable to classes only, as these do not exist anyway, not to real things: hierarchies of things, whatever they may be, cannot belong to logic, as logic is analytic and so empty or uninformative; it is a priori valid and so unable to handle the contingent reality. Still Russell was reluctant to use the fictitious character of sets as license, so that Landini's excuse for him, important as it is, will not suffice.

19. See previous two notes. As Russell stressed, assuming that abstract set theory is a sufficient basis of mathematics, the task of logicism is merely to reduce its axioms to logic. He reluctantly postulated some of its axioms—its axioms of infinity and of choice—as nonlogical. In addition he postulated his axiom of reducibility, even more reluctantly (p. 110). Meanwhile the situation has changed greatly and it is now much more problematic.

A theory of types distinguishes sharply between classes of things and classes of classes and classes of classes of classes, and so on, and postulates that all sets are stratified. (Thus the theory does not permit the construction of the set of all sets of members of a society plus its leader: it requires that in such a set we replace the leader by the set that contains the leader as a unique member.) The axiom of reducibility says that arithmetic is the same for all types (thus, $2 + 2 = 4$ whether these numbers are sets-of-sets or sets-of-sets-of-sets, and so on). The axiom of mathematical induction involves reducibility for any theory of types (p. 181).

discussion of language in his first book is too narrow.[20] But he never admitted failure.[21] Russell's failure was magnificent, and most commentators still consider his effort the best. This invites detailed study that should help develop a better broad outline, a more comprehensive view of the history of logic. It is here that Landini's discussion of Russell's unpublished *Theory of Knowledge* may very well signify.

Landini spends much effort correcting misreadings of Russell, and these largely relate to the theory of sensations and to the light that modern logic throws on it. He is very erudite and very faithful to the texts (too faithful: there is no taboo against commentators disagreeing with an author's own comments on his text, as long as they do so openly and respectfully), yet he ignores Russell's superb "Reply to Criticism" (Schilpp 1944).[22] Russell states there his last word on the matter, declaring (p. 704 there) that theorizing about sensations involves metaphysics. He nevertheless asserts that they always depict past situations and that they are not always true in every respect, much less certain. To this he adds (p. 717 there), "I think that almost everybody in the philosophical world disagrees with me on this subject, but I am quite impenitent, because I never find arguments brought against my logical atomism. I find only a fashion and a dogma." This raises the question, what is this logical atomism? His answer is clear enough to show that he considers atomic facts in the light of physics[23] at its

20. Wittgenstein, *Philosophical Investigations*, §93. His proposal to broaden logic to cover all language (here misascribed to John Austin, p. 249) is the most influential part of his teaching and of his popularity next to his dislike of all metaphysical discourse. This move sounds rationalist but it is the result of the loss of orientation. Aristotle developed logic as an instrument for debates and for proofs; Frege and Russell constructed logic as a rigorous basis for mathematics; Frege wisely ignored science and Russell was careful not to mix logic with epistemology or science. This is the chief characteristic of his attitude to logic and Landini rightly stresses and praises this characteristic. By contrast, Wittgenstein said, logic should be the study of the functions of language. This goes contrary to the traditional view of it as the theory of the descriptive part of language, the one that includes W. V. O. Quine's rejection of the view that any theory of modality is a part of logic. Wittgenstein devised this move—his definition of logic broadly—as his excuse for his reintroduction of metaphysics through the backdoor. This way, said Peter Winch, Wittgenstein made logic encompass all social science. (See Jarvie 1972, chap. 2).

21. See note 16 above. A long debate centered on the question, how much of the output of the young Wittgenstein did the older Wittgenstein endorse? Accounts of the famous poker incident show that he never withdrew his antimetaphysics.

22. Landini ignores this, as well as Russell's "Logical Positivism" of 1956 that he echoes poorly (p. 245).

23. The only viable version of logical atomism extant is Popper's (neglected) idea of relatively atomic statements (Popper 1959, 128, 130, 285–86, 379–81, 405).

best[24] (pp. 703–04 there). Wittgenstein took logical atomism to be idealist; Russell did not. He said (p. 718) that the assumption "only percepts exist" is merely "a technical hypothesis which I was trying to make logically adequate" and he declares his preference for his latest version of this idea. What should transpire after the dust is settled is that all his life Russell tried hard to oppose idealism by common sense, taking sensations to be both emanations from things and fairly (not fully) accurate information about them. Their limited reliability leads to epistemology (p. 718 there) and to Russell's *Human Knowledge*. This 1948 book turns out to be an attenuated version of the suppressed 1913 one that is at the heart of Landini's present study. What is most conspicuous in that 1948 book most is its advocacy of the theory of confirmation as probability. This theory is a fiasco: it cannot possibly reflect the limited reliability of observations. What is otherwise most conspicuous in that 1948 book is that it is an improved version of the British empiricist tradition. That tradition includes very few detailed studies, perhaps only those by Laplace and by Mach. Peculiar to Russell's version, however, is its fallibilism, the chief function of which is to fend off idealism.

This does not sit well with Russell's neutral monism, the idea that "only percepts exist," which, we remember, he said late in life, in error it seems, that he had not quite endorsed. Landini makes a point that Russell abandoned his 1913 grand theory partly because of "his conversion to neutral monism" (p. 66). The error that Wittgenstein criticized and this "conversion to neutral monism," he says, together made Russell stop writing that 1913 book. The criticism is not easy to repeat; the error is this: Russell viewed qualities, especially perceptible ones (like redness), as real and universal (= denoting all red objects). He wanted his language to include sentences with them not only as predicates (x is red) but also as subjects (red is a color). This leads to Russell's paradox. The previous sentence is not easy to show, as it requires an explicit statement of a formal system and a formal proof.

This is the crux of the development of logic in the period in question. At the time that Mach dismissed the liar's paradox as a minor matter, Frege realized that Russell's paradox was shattering. Later on Tarski offered a new solution to the liar's paradox that Landini uses here extensively. For naïve discourse Mach's position is still valid: there we can just ignore any paradox; not in formal systems: in an inconsistent formal system every statement is provable.

Russell's 1912–1913 manuscript that Landini discusses, in which Russell viewed classes as nonexistent, throws light on the theory of Wittgenstein,

24. Wittgenstein tried to be up to date regarding physics, but he was not (*Tractatus*, 4.11; 6.341-3).

who clung to this idea. It explains the import of Russell's theory of definite description (see below) that he was always proud of and which even Wittgenstein praised.

Let us take this slowly. Aristotle's logical system is one of terms, perhaps only of essential terms. (He excluded at one time or another contingent terms and proper names.) His logic gave way to Boolean algebra that is likewise a system of terms proper, be they proper names or class names, essential or accidental. Abstract set theory is so much richer because it contains classes of classes.[25] This immediately raises the question: how different are classes of classes of things from classes of things? Are class inclusion and class membership somehow comparable? Russell investigated the difference between them:[26] classes always include themselves as classes yet they do not contain themselves as members—at least not normally. His effort to prove that the nonnormal set is empty led him to his paradox. And as his aim was to develop logic as an axiom system adequate for mathematics, he had to get rid of all paradoxes. The first effort to develop such a system of logic is due to Frege. Russell did not know of him, but he knew of Peano who had held regular discussions with Frege. (Peano wished to increase rigor, so that his interest was more traditional than that of Frege and Russell. [Segre 1995]) Logicism, the theory that mathematics is a part of logic, was not new; the novelty was in the effort to prove it. This demanded the construction of a system of logic as a language that encompasses mathematical assertions. Diverse critics of logicism often overlook this accomplishment. Yet these critics have performed a difficult task too: they contributed to the present environment in which formal languages and formal systems proliferate. These serve each a specific goal, unlike systems of the early twentieth century, meant to reflect distinct worldviews. We had better ignore the proliferation here, on the understanding that this limits Landini's discussion to history.

Aristotle's system of logic is very limited; his claim for it was very high: he had captured the essence of language and of science: it is a set of terms and their classification. Boole's system that was so much broader came with no such pretense. It is very important for the comprehension of Frege's success that he eschewed the task of accommodating science. It is no accident that "Frege never articulated a detailed epistemology" (Notturno

25. Abstract set theory has many versions, and all of them allow for classes of classes and so on. Boolean algebras allow for classes of things only.

26. Russell reports that Cantor's paradox (the class of everything is bigger than itself) led him to consider this matter.

1985, 93): he was following Boole here (Bar-Am 2008). What part of language should the language designed for mathematics capture? Clearly, the calculus and geometry are in; clearly, ethics and esthetics are out. Should questions be in or out? This is a moot question, answered, if at all, by the logic of questions that evolved only in the second half of the twentieth century.[27] Nevertheless, this was clear: Frege, Russell, and Wittgenstein wanted the language in question to comprise a given fixed set of rules (grammar) and an open vocabulary (dictionary). And they wanted it to be rich enough to articulate any descriptive assertion, true or false (but not simultaneously so). This is not all: Frege wanted to capture the essence of the grammar of all human languages (p. 148). He said, if any language violates a given rule of grammar, then that rule is out. Since clearly the structures of sentences in the new system of logic were mathematical, the new language must differ from all known natural languages. How then is it supposed to reflect them? Come to think of it, what is the structure of a sentence that it shares with its synonyms in other languages? Worse, there are synonymous sentences within one given language, are they all of the same structure? At times they are: "a precedes b" and "b succeeds a" are synonyms comprising one sentence in the Frege–Russell system. Is this always so? Surely not: every sentence is synonymous with its double negation, yet the structures A and ~~A differ. This is puzzling.

Readers of Russell's *Principles of Mathematics* of 1903 must have noticed that he was struggling with these questions in addition to all the standard questions that occur in logic. His solution to the paradox offered him the way. He was concerned with other paradoxes, including the following: the king wanted to know who the (anonymous) author of *Waverley* was; the author of *Waverley* was Walter Scott; but the king did not want to know who Walter Scott is (since he knew him). Russell overcame this paradox by spelling out carefully each of these sentences (1905). This is his celebrated theory of definite descriptions.[28] To justify this he said, we all do the same all the time. He gave an example: I thought your yacht is bigger than it is. (Another example: the time you enjoy wasting is not wasted time.) Taken literally, this sentence is absurd, yet normal speakers of the language will not see the absurd here, as they will reinterpret the sentence to mean what

27. See Agassi (1975, chap. 10: Questions of Science and Metaphysics).

28. The word "definite" comes from the fact that Russell explained "Sir Walter Scott is the author of *Waverley*" as "there exists exactly one person whose name is Sir Walter Scott and who is the author of *Waverley.*" It was soon read too narrowly–as a definition of the definite article. Landini avoids this error.

the person who uses it means it to say, not what it literally says. This way Russell discovered what later researchers called syntactic ambiguity: to understand a sentence is to observe its syntactic structure (which is often veiled) and in this background knowledge may play a crucial role.[29]

It is thus not surprising that the studies of Frege and of Russell that pertain to mathematics have a different status than their studies of all descriptive language. The study of the structures of sentences in diverse languages and their preservation through adequate translations from one natural language to another[30] is still in its infancy, as the study of the structures of sentences in any one given natural languages is still too hard.[31] Yet this much we owe to Russell: the rules of grammar that we learn in school are obsolete; the structures of sentences are not self-evident.[32]

This was the state of affairs when Russell embarked on his grand project that is the heart of this book (and of its predecessor). Already Frege's theory of variables and quantifiers over them did much to clarify syntactic ambiguity. Russell offered a theory for it. He wanted the syntax to reflect the structure of our knowledge of the world and the structural part of the world that logic has to reflect. This is impossible. Take a simple point. Russell wanted truth to be a characteristic of true sentences, not of vacuously true ones (pp. 46–49), and of elementary ones first. Yet conditional sentences whose antecedents are not true are indeed vacuously true. (Translating a conditional to a disjunction will not help here.) And this quite apart from the fact that there is no criterion as

29. Today linguists take the idea of syntactic ambiguity in their stride, and many of them follow a rule of thumb of Noam Chomsky: every statement whose structure is veiled is logically equivalent to one with an obvious structure that unveils the veiled structure. Thus, to differentiate between two homonymous sentences that are not synonymous one can translate them to their structure-revealing synonyms. Thus, a sentence that describes the shooting of the hunter will be equivalent to ones that describe the hunter once as active and once as passive, thus revealing the two different structures of the two look-alike sentences. Chomsky offers examples where decision as to the structure of a sentence unconsciously rests on some background knowledge: we assume that a person involved in cooking is active rather than passive, unless we speak of cannibalism.

30. A sentence retains its semantic ambiguity in translation from one natural language to another; the purpose of translation to formal language is to get rid of ambiguity.

31. See Agassi (1997).

32. As Landini notes (pp. 12, 227), on this Wittgenstein was full of praise: "It was Russell who performed the service of showing that the apparent logical form of a proposition need not be its real one" (*Tractatus*, 4.0031). Landini finds this an exaggeration. He claims that analysis separates everyday concepts form their scientific cognates. To show this he offers his own analysis of a sentence that requires none, and does a poor job of it by a poor presentation of a common scientific term (p. 227).

to what fact is elementary and so no criterion as to what statement is elementary. At the time Russell advocated logical atomism and suggested—following Wittgenstein—that atomic propositions reflect elementary facts. He withdrew this idea but held some variant of it, we remember. Perhaps he did not quite know what it was exactly. Landini quotes (p. 70) his (Russell's) famous letter to Lady Morrell (1916) in which he said he was still "convinced that all fundamental work in philosophy is logical." This is logical positivism at its very best (no trace of meaning analysis here). To take it on faith, however, was nothing short of theft; to struggle with it in detail was the honest toil that exhausted Russell. He could not but dismiss Wittgenstein's output as facile. Landini gives us the taste of this ever so heavy and grinding toil as evidence for the poverty of the view of Wittgenstein as superior to Russell as well as of the view of Russell's work that makes little of his enormous toil.

Yet the advantage of theft over honest toil signifies. Some theft is no more than unfair benefit at others' expense. Some theft, however, is covert borrowing of something to put to good use, with a covert promissory note. At times, however rare, a thief does make good and returns the debt with handsome interest. This does not make theft moral or advisable, but it is an advantage all the same, even if a thin silver lining of a very heavy dark cloud. All the more so for property that is potential and intellectual: researchers always write such promissory notes.[33] So the jobs they undertake are risky, often disappointing, and then the researchers and their associates have to bear the loss. Landini deems Russell's admission of Wittgenstein's critique to be an admonition to work ever-harder (p. 71):

> Wittgenstein accused Russell's work in his *Theory of Knowledge* as betraying the research program of logical analysis. . . . Russell had forged ahead with the work, working at a remarkable speed of ten pages a day. His enthusiasm for work is clear. But the pace found him with promissory notes (of the availability of a more exacting analysis) when dubious assumptions arose (such as the identification of logical forms with abstract general facts).

Later Russell viewed the advantages of Wittgenstein's output to be mere theft (p. 226).

Russell's theory of definite description came to replace Frege's theory of sense and reference. Frege had refuted both nominalism and realism by

33. To be precise, practically all past intellectual success rested on questionable promissory notes of this kind. This is very much in line with Russell's view of all logic and mathematics prior to Boole as all too shoddy.

refuting the idea they share, that meaning is denotation or reference or pointing at the object that a name names. Roughly speaking, by his alternative theory a name refers to two objects, one in space-time and one out (in a Platonic heaven). Russell's theory of definite description was a partial success in that it allowed for naming and describing with reference only. This was but a first step. He then tried to develop a theory that did not denote (did not postulate the existence of) any logical entity, any class, any proposition—any abstract entity of any sort. This was his grand project; he failed and admitted defeat in 1940 (*Inquiry into Meaning and Truth*).[34] Yet much earlier Wittgenstein had declared all this toil unnecessary: it was enough to declare that only atomic facts are real and leave it at that. This, observed Russell, left the logicist program in mid-air: there still was mathematics to take care of, and Wittgenstein had not delivered the goods, not even a promissory note.

Russell both fought Frege's Platonism in 1905 and found his partial success monumental. Yet he still had a mystic approach to mathematics that Wittgenstein cured him of, a cure for which he expressed gratitude most reluctantly. This does not make good sense. Landini cites Monk (p. 73) to say that under the influence of Wittgenstein Russell "regarded mathematics not as a body of knowledge but as a set of tautologies" akin to "all four-legged animals are animals." Landini says outright, "This is a serious misunderstanding." He cites (pp. 158–9 and nn. 17–18 there) clear evidence that Wittgenstein denied vociferously ever having held this view. The story that we tell our students is that logicism is the effort to show that all mathematical truths are logical, namely [!] tautological, and here we see Russell suffering as Wittgenstein convinces him of this truth, a truth that makes him lose the joy of life, a truth that Wittgenstein denied that he had ever held! If this is not confusing, nothing is.

Landini explains the situation (chap. 4). Russell took logical truth to be truth by virtue of form, which we now call tautologous. He tried to render all mathematics such truths and failed. "Wittgenstein was eager to offer an escape" (p. 110). Here (p. 111) Landini quotes from Russell's 1927 text the

34. The admission there is of a failure to eliminate Platonism. The view that the axiom of infinity is essential for mathematics and applies to real objects amounts to the assertion that mathematics requires infinitely many real objects. It is an admission of defeat. The way out is to notice that as parts of logic, axioms refer to potential objects (or even to possibly real abstract objects). This move did not satisfy Russell: he viewed it as clandestine Platonist. Now we have to assume the existence of abstract objects anyway, but, Russell hoped, only as fictitious. This idea is the basic aspect of the ontology of Mario Bunge (1974). Bunge claims to have ousted Platonism successfully. Since his logic is more in the Polish than in the logicist tradition, his ideas have not yet gone through the critical scrutiny that they amply deserve.

statement that logical truths are tautologies, and he ascribes it to Wittgenstein. I confess this puzzles me, as this is the thesis of logicism that Russell was intent on proving long before the appearance of Wittgenstein on the scene. Russell's text continues with the assertion of logicism and the rider that Wittgenstein might dissent from it. Clearly, Wittgenstein made a serious contribution here that Russell found necessary to acknowledge. What is it, then? Landini is aware of the problem here: he observes that in the Introduction to the second edition of his *Principles* of 1937 Russell says a tautology is "true by virtue of its form," yet with no acknowledgment to Wittgenstein. That this is problematic is clear: Russell says this is "obvious" yet he confesses he cannot say exactly what it means. Landini responds with "Russell seems to back away from his earlier enthusiasm for Wittgenstein's idea" (p. 112). "Seems" is the operative word: for a judicious reading we need more details. These are intricate: there is a seeming agreement between the two, but only a seeming one. "Wittgenstein was eager to offer an escape," we remember; Russell could take his idea seriously if and only if he offered something that at least looked like an escape. We could thus suggest that Russell took Wittgenstein's idea seriously at first as a possible escape and changed his mind when it turned out to be a dead-end. This invites detailed information. Here it is.

Wittgenstein was looking for a notational system that would render obvious the tautologous character of a tautology. This idea looks today way-out, not to be entertained seriously. This is a mistake of hindsight. It makes excellent sense to take it seriously as a pivotal aspect of the philosophy of the young Wittgenstein and it is a revelation for which Landini has my gratitude.[35] It explains why Russell gave as an example for a tautology the sentence "all four-legged animals are animals." Assume that mathematics is logic and that all tautologies are as boring as this sentence is. This insures the destruction of the beauty of mathematics that meant so much to Russell. This explains why considering Wittgenstein's idea true depressed him profoundly. This explains what Wittgenstein said that was no part of logicism. Yet we have to see what this may—only may, remember—have to do with Russell's failure to prove logicism from which "Wittgenstein was eager to offer an escape." Clearly, when Russell said in 1937 that he did not quite know what truth by logical form means, he meant that he could not see whether the rescue of logicism was serious or not.

35. Landini finds (p. 112) the idea fully expressed in Wittgenstein's *Tractatus* (6.1265ff.). God knows how often I have overlooked this and similar statements in that opaque book while brooding on it—until I read Landini.

The crux of Landini's book (chap. 4 and appendix A) concerns Wittgenstein's effort to prove the axioms of quantification theory (the theory for the operators "all" and "some") with identity (pp. 116ff.). This is a minor trouble for Russell;[36] for Wittgenstein it was a very big one, since quantification does not yield to truth tables and since identity is central for the logicist idea of arithmetical equations as identities.[37] So we can understand his being "captivated" (p. 117) as he thought he found that his notational idea renders the quantification axioms obvious tautologies and seemingly does away with identity.[38] In 1936 a disproof of this appeared

36. Russell repeatedly admitted as a minor trouble with his system that it implies that the world is nonempty. Quine showed a way to overcome this difficulty (p. 225). Another way is that of Hughes Leblanc (1995).

37. The view of arithmetic as a set of identities is still popular; Tarski advocated it. Strangely, this idea that Wittgenstein vehemently opposed has a Wittgensteinian flavor: to say that 2 + 2 is another name for 4 is to say that the first name tells us more about its construction; 1 + 1 + 1 + 1 says still more, and 0'''' is then the best in principle. But it is terrible when it comes to larger numbers. (In that case we enumerate the apostrophes rather than write them down, and then we are back to square one.) This shows how limited Wittgenstein's idea is. In the same way, incidentally, it is possible to view the common name for a conjunction, for example, as an indication of how we usually construct it: we write the words of one sentence, and then a word of conjunction and then the words of the other. Of course, this is not necessary: we can write the conjunction sign at the end, as in Latin and we can use shorthand, so that "p" is shorthand for a sentence that we call *a*. It is hard to choose between the names "conj (*a, b*)" and "*ab,*" as one is better for one end and the other for another. This is the limit to Wittgenstein's grandiose idea. See Karl Popper (1947).

38. "Russell admitted that for a time he accepted Wittgenstein's view on identity but soon came to the conclusion that it made mathematical logic impossible." (p. 220). Ramsey underwent a similar process. The view of the logical operators that Wittgenstein offered in his first book seems extremely problematic. It is not, popular opinion to the contrary notwithstanding; his proposal is simplistic: he suggested that since the word "all humans" refers to all and every human being, there is no problem with "all humans are mortal" and that this should suffice. (It amounts to the omission of the universal operator and taking it as said, which is an accepted procedure.) Now his simplistic view does not work, and so commentators rightly reject it, but this is an elaboration on a simple idea that just happens not to work in sophisticated logical systems. Wittgenstein opposed the sophistication. Wittgenstein, in my naïve opinion, made a naïve observation and it is best to see it as (erroneously) naïve and leave it at that as far as exegesis is concerned. Russell and Ramsey, however, were no exegetes, and so their effort to make better sense of Wittgenstein's proposal is a different matter: it is one thing to take a proposal seriously and quite another to assume that its author made it seriously. Let me note, however, that identity is still problematic. (See pp. 222-3). Tarski proved *(Introduction to Logic)* that it is not a logical sign: the statement that all things are identical to each other is true only for an empty universe and for one with a single object. The best on it, however, is Quine (1987).

(p. 118)[39] and freed Russell from the need to make acknowledgement to Wittgenstein. Today the inadequacy of Wittgenstein's notational idea wins general recognition effortlessly (end of chap. 4).

This story is very thin. From quantification theory to the axiom of infinity a logicist will find immense distance. This explains the tenor of Russell's Introduction to Wittgenstein's first book. He ignored quantification and came down hard on Wittgenstein's neglect of infinity. Eager to offer an escape as Wittgenstein was, he offered none, not even a poor one. So the picture is not very thrilling. Still, it is the best picture we have of a very complex train of events.

Wittgenstein was in error when he hoped to "offer a notation which realizes the goal" of revealing logical structure. He was "eager to offer an escape" for logicism and did not. He did not "even attempt to offer" such a notation (p. 156). Was he a logicist? In Frege's and in Russell's senses, surely not: he wanted logic to *show* itself. Can this be a form of logicism? Landini devotes his chapter 5 to the affirmative answer: Wittgenstein was a logicist in two senses. One sense is this: he did offer a way to avoid the use of the axiom of reducibility[40] for the establishment of the axiom of mathematical induction (pp. 182, 202), and Russell proved him right on this; for logicism this is insufficient, but it goes the logicist way (pp. 191, 211ff.). Uncharacteristically, Landini does not elucidate this; all he says is, Wittgenstein hoped to prove that the principle of mathematical induction is redundant. So we see here again a person who hates promissory notes writing a big one. The second sense in which Wittgenstein was a logicist is this: he viewed both logic and mathematics as sets of rules for calculation (p. 185). As rules are not statements, we remember, Wittgenstein considered them meaningless. Now science involves calculations too; hence, his reduction to logic is not only of mathematics but also of science. Already Ramsey launched this criticism (p. 186). Wittgenstein answered him in his first book (still p. 186). I do not see how this answer makes Wittgenstein a logicist. He rejected Frege's and Russell's view of arithmetic and developed a new version of it, and a new assessment of it—as equations rather than tautologies. (No, I do not understand this contrast and Landini does not explain it.) Russell viewed Wittgenstein's version as finitist; Landini finds this surprising. The matter is controversial. It seems to me undecidable, at

39. The disproof in question was the 1936 article by Alonzo Church. Already the 1932 article by Kurt Gödel threw severe doubt on the idea, but Church's article was a deathblow—to the whole of Wittgenstein's system (Bartley, 1973, pp. 70-79).

40. See note 19.

least in the light of the lack of clarity about Wittgenstein's view of mathematics as tautologous. He was too shrewd to fit any standard categorizing, and so he could honestly deny any attribution to him of any philosophical view whatsoever. Nevertheless, he could not evade all criticism. His view of his arithmetic as decidable is false—unless one insists that it is finitist and that finite arithmetic is decidable. (For this see p. 187.)

The final chapter of the book deals with an important question that is still open. Since logic has to leave questions of fact open as much as possible,[41] what factual matters are impossible for logic to avoid?[42] The chapter is disappointing. It discusses mainly Wittgenstein's influence on Carnap and cites a lovely article by Alberto Coffa that shows the development of a crack in the doctrine of showing in that Wittgenstein (and Carnap) allowed for "strange sentences" (pp. 245–46). Bar-Hillel had argued earlier that there is no grammar to deal with them. Thus Carnap could declare such sentences ("this stone is now thinking about Vienna") once meaningless and once false (Bar-Hillel 1964, 34).

The task that the early literature on logic tackles, Landini says, is even more ambitious than that of uncovering the ideal language: it is to transcend language and say something about it. That this is impossible is all too obvious. Yet Frege found it once necessary to do so (p. 79ff.) and he did so "with a pinch of salt." Russell objected to this and Wittgenstein made this a cornerstone of his first book. The result was that he had to state vital logical information in pseudo-propositions (p. 82). This is pathetic, and Russell criticized it mercilessly in his Introduction to Wittgenstein's first book. As Landini notes (pp. 83, 152, 154), the inspiration for Wittgenstein was none other than the Introduction to the *Principia* that contains important logical information that is not a part of the system. (It belongs to the meta-language; Russell first used this concept in his Introduction to Wittgenstein's book.)[43] Wittgenstein's *Tractatus Logico-Philosophicus* is a highly simplified version of *Principia Mathematica* of Whitehead and Russell. This is hardly news. Russell destroyed the system of that book. This too is hardly news. Thus far progress could not put it together again (p. 247).

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41. We obviously must demand of logic to leave open those questions that we want to discuss logically (Popper).

42. This question is not pressing as the choice of assumptions common to parties to a discussion is always open. Russell and Wittgenstein wanted the ultimate philosophy and the ultimate logic; this is impossible. Russell noted that to claim freedom from all prejudice is humbug; suffice it then that from time to time we can get rid of some.

43. Brouwer and Hilbert used the concept of the meta-language earlier (and had a priority dispute). Wittgenstein never withdrew his refusal to use it. A. J. Ayer (1985) suggested that he consequently refused to take the paradoxes seriously. This is an error: Wittgenstein assumed that proper grammar, whatever it is, dissolves them, as his notation will one day *show*.

Chapter 3 is an exegesis on Wittgenstein's views on ethics and on religion: Landini argues that early Wittgenstein was Russellian. He endorsed Russell's attitude to religion and ethics. He shows that Wittgenstein endorsed Russell's neutral monism[44] and its rationale.[45] This, however, Wittgenstein could hardly assert explicitly, and his first book barely touches on these matters, as his doctrine of ineffability forced him to be silent on them: contrary to popular readings, he revered metaphysics; he opposed only its articulation: one can show, not assert, the truths of logic and of metaphysics: the elect, the mystic teacher, only points the way.[46] And so, a vast difference between Russell and Wittgenstein hardly received attention.[47] Indeed, here is the most important case of agreement for different reasons: skepticism about logic is impossible: Russell said it is irrefutable but unreasonable, Wittgenstein said it was meaningless (p. 104).[48] All this is very interesting and its link with logic as both Russell and Wittgenstein saw it is very helpful for the understanding of the goings on. Russell was extremely frustrated as he conceived his positivism and then he found a few young students who understood him and even chipped in and participated in working out the details of the exercise. Russell admired their ability to partake in honest toil and declared one of them—Wittgenstein—his heir.[49] He was bitterly disappointed. Soon Wittgenstein had a huge following, most of them mathematically uninformed[50] and not interested in Russell's

44. Popper (1945, ii, chap. 11, Notes) understood Wittgenstein's philosophy as neutral monism. Popper's reading of Wittgenstein first created a wave of protest and then effortlessly turned official.

45. The rationale for Russell's neutral monism was both epistemological and logical. Surprisingly, he dodged it as metaphysics. He said, however, he posed his neutral monism as a hypothesis ("Replies," p. 718), and if reconciling it with commonsense realism is impossible, he preferred to be viewed as a commonsense realist rather than as a neutral monist. Not so Wittgenstein (*Tractatus*, 5.62-64).

46. Russell was the first to see Wittgenstein as a mystic, and for a long time he was alone on this. Landini discusses at length Wittgenstein's doctrine of showing, but not its being a version of the mystic doctrine of pointing the way.

47. Russell was always under the influence of his grandmother's Puritanism and his grandfather's sense of public duty. He later became a hedonist, as he describes in his *Autobiography*. Wittgenstein was a Tolstoyan of sorts but with no sense of public duty. And he left no room for debates on matters of ethics. The same holds for Ayer, who differed in ethics from Wittgenstein. Their proscribing a debate on ethics is thus appalling.

48. Wittgenstein said in explanation, a question with no answer is impossible (*Tractatus*, 6.54). This is the basis of much discussion that Wittgenstein's disciples in the mid-century found exciting. It is of course most unreasonable: Wittgenstein had no theory of questions. Little has remained of these discussions. (See also note 10 above.)

49. Russell was very fortunate with his teacher—Alfred North Whitehead—and very unfortunate with his brightest students: Jean Nicod and Frank Ramsey died young and Wittgenstein disappointed him.

50. See Russell (1958).

ideas. He admitted he found it unpleasant. Meanwhile things have changed and a more balanced view emerges.

## References


Agassi, J. 1975. *Science in flux.* Dordrecht: Kluwer.

———. 1988. The secret of Carnap. In *The gentle art of philosophical polemics***.** La Salle, IL: Open Court.

———. 1997. The novelty of Chomsky's theory. In *The future of the cognitive revolution,* edited by D. M. Johnson and C. E. Enerling, 136–48. Oxford: Oxford University Press.

———. 2000. To close the Wittgenstein project. *Iyyun: The Jerusalem Philosophical Quarterly* 49:313–29

———. 2006. On proof theory. In *Is there certain knowledge?/Gibt es sichere erkenntnis*? edited by M. Rahnfeld, 264–82. Grundlagenprobleme Unserer Zeit, Band V, Leipzig: Leipziguniversitätverlag.

Ayer, A. J. 1985. *Wittgenstein*. London: Weidenfeld & Nicholson.

Bar-Am, N. 2008. *Extensionalism: The revolution in logic*. Berlin: Springer.

Bar-Hillel, Y. 1964. *Language and information: Selected essays on their theory and application.* Reading, MA: Addison Wesley.

Bartley, W. W., III. 1973. *Wittgenstein.* New York: Lippincot.

Black, M. 1964. *A companion to Wittgenstein's Tractatus*. Ithaca, NY: Cornell University Press.

Bunge, M. 1974-89. *Treatise on basic philosophy*. Dordrecht: Reidel.

Hintikka, J. 1996. *Ludwig Wittgenstein: Half-truths and one-and-a-half-truths*. Berlin: Springer.

Jarvie, I. C. 1972. *Concepts and society*. London: Routledge.

Leblanc, H. 1995. On axiomatizing free logic—And inclusive logic in the bargain. In *Québec studies in the philosophy of science,* part I: *Logic, mathematics, physics, and the philosophy of science*, *Boston studies in the philosophy of science,* vol. 177, edited by M. Marion and R. S. Cohen, 1–22, Dordrecht: Kluwer.

Notturno, M. A. 1985. *Objectivity, rationality, and the third realm: Justification and the ground for psychologism: A study of Frege and Popper*. Dordrecht: Kluwer.

Popper, Karl. 1945. *The open society and its enemies*. London: George Routledge & Sons.

———. 1947. New foundations for logic. *Mind* 56:193–235.

———. 1959. *Logic of scientific discovery*. London: Hutchinson.

Quine, W. V. 1987. *Quidditiies*. Cambridge MA: Belknap Press.

Rodych, V. 2007. Wittgenstein's philosophy of mathematics. *The Stanford encyclopedia of philosophy.* http://www.seop.leeds.ac.uk/entries/wittgenstein-mathematics/.

Russell, B., 1950. "Logical Positivism", in his *Logic and Knowledge,* 1956, pp. 367-82.

———.1958. Mathematical infinity. *Mind* 67 (July): 385.

———. 1959. *My philosophical development*. London: Allen & Unwin.

Schilpp, P. A., ed. 1944. *The philosophy of Bertrand Russell,* vol. 2. New York: Harper's Torchbooks.

Segre, M. 1995. Peano, logicism, and formalism. In *Metaphysics and science: Essays for Joseph Agassi,* vol. I, edited by I. C. Jarvie and N. Laor, 133–42. Dordrecht: Kluwer.


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